Seventh Annual Upper Peninsula High School Math Challenge

Northern Michigan University (Marquette, MI, USA)
Saturday 12 March 2016

TEAM: _____

SCHOOL: _____

TEAM PROBLEMS   **TIME: 45 minutes**

1. _____

2. _____
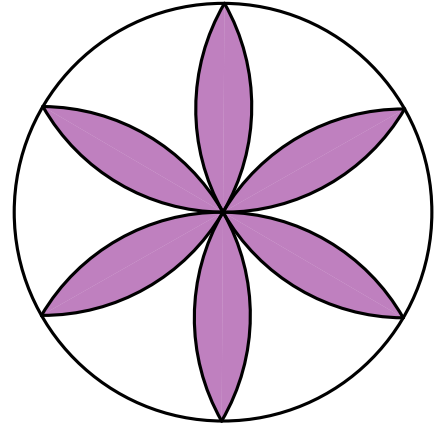
3. _____

4. _____

5. _____

Put no work on this side of the paper. Write the answers <u>only</u> in the above spaces.

Put all work on the enclosed sheets of scrap paper, and hand in the scrap paper with your answer sheet.

1. A "regular hexaflower" is inscribed in a circle of radius 2 units. Find its area.
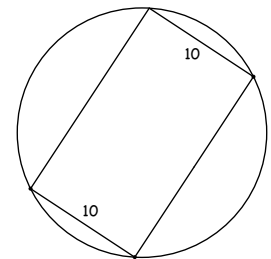
    Express your answer in terms of π and/or radicals (if appropriate). No decimal approximations.

2. Find all ordered pairs of real numbers (x, y) that satisfy the equations

$$3^x \cdot 9^y = 81 \quad \text{and} \quad \frac{2^x}{8^y} = \frac{1}{128}.$$

3. In a circle of radius 10 cm, two parallel chords equal in length to the radius form opposite sides of a rectangle. What is the area of the rectangle?

4. Let f(x) = ax + b. Find all real values of a and b such that f(f(f(1))) = 29 and f(f(f(0))) = 2.

5. △ABC is a right triangle with segments AD, DF, FE, EC, and CB all of equal length. Find the measure of ∠A in degrees.